Commun. Math. Phys. 117, 79–108 (1988)

Communications in Mathematical Physics

Renormalization of Feynman Integrals on the Lattice

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Abstract. A perturbative renormalization procedure is proposed which applies to massive field theories on a space-time lattice and is analogous to the BPHZ finite part prescription for continuum Feynman integrals. The renormalized perturbation theory is shown to be universal, i.e. under very natural assumptions the continuum limit exists and is independent of the details of the lattice action.

1. Introduction

In perturbation theory of a local quantum field theory there exist well-known renormalization procedures which remove ultraviolet divergencies. The BPHZ finite part prescription makes subtractions directly in the integrand of each Feynman integral in momentum space [1]. Divergencies of every subdiagram are subtracted by application of a Taylor operator in the external momenta of the subdiagram, which in position space is a local operation. The renormalized Feynman integral is defined in such a way that the ultraviolet (UV) divergence degrees of all subdiagrams are negative. There exists a power counting theorem due to Hahn and Zimmermann [2], which states the convergence of integrals having this property.

Unfortunately, these methods assume a rational structure of the Feynman integrands and hence do not apply to diagrams with a lattice cutoff. In this case, instead of being rational, the integrand is periodic with the Brillouin zone. Removing divergencies by subtraction of Taylor polynomials is very unnatural in a lattice description, and in fact such a procedure does not work, due to violation of periodicity. In a recent paper [3], we have proposed a lattice version of the power counting theorem of Hahn and Zimmermann by generalizing the well-known notion of a UV-divergence degree. Having such a theorem at our disposal, we are able to construct a generalization of the BPHZ finite part prescription which applies to diagrams with a lattice cutoff. Due to power counting conditions, the combinatorics of subtractions are given by the forest formula of Zimmermann [1]. As will be seen, the important modification consists in replacing Taylor operators