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Sigma Model Solitons and Their Moduli Space Metrics

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Abstract. We discuss the supersymmetric σ model and its soliton solutions in $2+1$ dimensions. We classify supersymmetric maps and derive Bogomolny bounds. We also give the modified superalgebra and describe the metric on the parameter space of solitons.

Introduction

Nonlinear σ models have been studied for many reasons. In 2-Euclidean dimensions analogies have been considered with 4-dimensional gauge theories, whereas in Lorentzian spacetimes the supersymmetric and extended supersymmetric σ models have been discussed, because of their finiteness properties and its relation to complex geometry. More recently σ models defined on Riemann surfaces have been considered in string theory where the Riemann surface represents the world sheet of the string propagating through spacetime.

In this paper we wish to discuss the two dimensional instanton solutions and some metrics associated with them. The primary motivation was to understand the work of Ward [1] and of Zakrzewski and collaborators [2, 3] in a wider context. These authors have discussed the evolution of solitons in the $2+1$ dimensional CP^1 and CP^n models. A technique which has also recently been applied to similar problems is that of the approximation of geodesic motion on a moduli space [1, 4–6]. That is at low energies in a given topological sector the evolution of solitons may be approximated by a motion on a finite dimensional submanifold of an infinite dimensional configuration space. On this submanifold the evolution is given by a geodesic motion with respect to a natural metric and for the CP^n models the submanifold represents the instanton solutions in 2 Euclidean dimensions. Zakrzewski and collaborators [2, 3] found that for the CP^1 model these natural metrics were formally Kähler. Here we generalize this result to a wider class of target manifold and elucidate properties of the metrics already found.

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