Commun. Math. Phys. 115, 649–690 (1988)

Communications in Mathematical Physics

$C=1$ Conformal Field Theories on Riemann Surfaces

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Abstract. We study the theory of $c=1$ torus and $\mathbb{Z}_2$-orbifold models on general Riemann surfaces. The operator content and occurrence of multi-critical points in this class of theories is discussed. The partition functions and correlation functions of vertex operators and twist fields are calculated using the theory of double covered Riemann surfaces. It is shown that orbifold partition functions are sensitive to the Torelli group. We give an algebraic construction of the operator formulation of these nonchiral theories on higher genus surfaces. Modular transformations are naturally incorporated as canonical transformations in the Hilbert space.

1. Introduction

Both in the study of two-dimensional critical phenomena and in string theory one of the major goals is to find a complete description of all conformal field theories [1]. In the first context these describe all universality classes of critical models [2], while in string theory they are known to correspond to all possible compactifications [3]. The conformal field theories with central charge c less than 1 have been successfully classified. The combined constraint of unitarity and one loop modular invariance selects a discrete set of models [4]. The extension of the analysis to c values larger than 1 appears to be a much harder problem and will surely reveal a quite different structure. For $c \geqq 1$ there is the possibility of a continuum of inequivalent models, since marginal operators can be present in the spectrum that generate continuous deformations of the conformal field theory.

In [5] an elegant formulation of two-dimensional conformal field theory has been given in terms of the behaviour of the partition function on the space of all inequivalent compact Riemann surfaces. The basic characteristics of the theory are translated into consistency conditions on the partition function, defined as the hermitian norm on a flat holomorphic vector bundle over this moduli space. The correlation functions are obtained through factorization of the partition function at the boundary of moduli space. Crossing symmetry of the amplitudes is then a consequence of modular invariance.