Commun. Math. Phys. 112, 567–590 (1987)

Communications in
Mathematical
Physics

Analytic Fields on Riemann Surfaces. II

V.G. Knizhnik

Landau Institute for Theoretical Physics, Moscow, USSR

Abstract. The properties of analytic fields on a Riemann surface represented by a branch covering of $\mathbb{CP}^1$ are investigated in detail. Branch points are shown to correspond to the vertex operators with simple conformal properties. As applications we compute determinants of $\bar{\partial}_j$ operators for Z_n-symmetric surfaces and obtain various representations for the two-loop measure in the bosonic string theory together with various identities for theta-functions of hyperelliptic surfaces. We also present an integral representation for the quantum part of the twist field correlation functions, which describe propagation of the string on the orbifold background. We also calculate the quantum part of the structure constants of the twist-field operator algebra, generalizing the results of Dixon, Friedan, Martinec, and Shenker.

1. Introduction

The chiral pairs of anticommuting fields in two dimensions are known to play an important role in string theory and in conformal quantum field theory. Recently it was shown [1, 2] that the conformal theory of such fields can be constructed on an arbitrary Riemann surface with the special singular metric, under a careful account of all anomalies and zero modes. In the present paper we will continue the study of analytic fields, but now it will be more convenient for us to represent the surface as a branched covering of $\mathbb{CP}^1$.

Let us denote by Z the covering map of a surface X on $\mathbb{CP}^1$:

$$Z: X \to \mathbb{CP}^1, \tag{1.1}$$

and choose the metric on X to be

$$g_{zz} = g_{\bar{z}\bar{z}} = 0, \quad g_{z\bar{z}} = 1 \tag{1.2}$$

the complex structure of X being induced from $\mathbb{CP}^1$.