Commun. Math. Phys. 112, 205–216 (1987)

Communications in
Mathematical
Physics

Generalization of Sturm-Liouville Theory to a System of Ordinary Differential Equations with Dirac Type Spectrum

Chen Ning Yang

Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794, USA

Dedicated to Walter Thirring on his 60th birthday

Abstract. The Sturm-Liouville theory is generalized to Dirac-equation-like systems of ordinary differential equations. It is shown how the comparison theorem and conversion to integral equations can be generalized.

1. Introduction

Some time ago [1] it was found necessary to generalize the Sturm-Liouville type of comparison theorems to coupled equations with a Dirac-like spectrum. The Sturm-Liouville theory deals with Schrödinger-like equations whose eigenvalues are bounded from below, i.e., equations which follow from a variational principle:

$$\delta \int (\psi, H\psi)dx = 0$$

for normalized ψ's, where the integral is positive definite. Elegant comparison theorems in the Sturm-Liouville theory allow one to have powerful information on the number of eigenvalues, on the nodes of the wave functions and on the meaning of Levinson's theorem. Furthermore by converting the Sturm-Liouville problem to that of an integral operator with a symmetrical kernel, one has powerful control over properties of the eigenfunctions and eigenvalues.

In the present paper we shall show that all these can be generalized for a large class of Dirac-type equations in one variable, for which the eigenvalues extend to both $+\infty$ and $-\infty$. Use has already been made [1, 2] of these generalizations. The detail of the generalizations is published here for the first time.

The results of the present paper can be easily further generalized. E.g., one could deal with an Hermitian potential $V(x)$ rather than a real symmetrical one. One could generalize the matrix ω of (2.2), etc. No such generalizations are attempted in the present paper.