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The Quantum Hall Effect for Electrons in a Random Potential

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Dedicated to Walter Thirring on his 60^th birthday

Abstract. We assume the existence of sufficiently localised states, near the edges of each Landau band. We then prove that the Hall conductivity is quantised and the parallel conductivity vanishes, when the filling factor stays close to an integer. The Hall integer is a topological invariant, given by the Landau band index.

We also prove that at weak disorder, the localisation length diverges in each Landau band.

0. Introduction

Since the experimental discovery of the integer quantum Hall effect, by von Klitzing [1] in a system of electrons confined to move in a plane, there have been many theoretical attempts to explain it on a microscopical basis.

The most famous one has been that of Laughlin [2], who, on the basis of a model independent argument, concluded that the quantisation should hold quite generally, when the electronic chemical potential lies in a gap or in the region of localised states. Later on, Halperin [3] showed that Laughlin's argument leads to the conclusion that the localisation length should diverge at least for some energies, when a strong magnetic field is present, in contrast to what is presently believed to occur in the absence of a magnetic field, for a two-dimensional system.

The first proof of Laughlin's conjecture in a model was made by Thouless et al. [4], (TKNN), who considered a system of non-interacting electrons in a periodic potential and a "rational" magnetic field, the electronic chemical potential lying in a gap. Avron et al. [5] soon proved that the TKNN integers associated to the Hall conductivity had a topological interpretation. (To a certain extent the topological aspect of the problem was anticipated in the not so well-known work of Dubrovin and Novikov [6], and Novikov [7] who considered mostly the case of a periodic magnetic field.) Later on it was realised by a number of authors: Niu and Thouless [8, 9], Avron and Seiler [10, 11], Tao and Haldane [12], Grümm et al. [13] that the analysis of TKNN did not really need the consideration of non-interacting electrons, but could be applied to any finite system of electrons, confined to move