

# Completely Integrable Models in Quasicrystals

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**Abstract.** The general method of construction of integrable dynamical models in quasicrystals is presented in the paper. It is illustrated on the example of the model of interacting spins for Penrose nonperiodic tiling of the plane. Another example constructed is the three dimensional model of interacting spins for icosahedral tiling of the three dimensional space. The bulk free energy is calculated for these models.

## 1. Introduction

The interest for nonperiodic tilings came first from problems of mathematical logics [1, 2]. However, since the invention by Penrose of his well known aperiodic tilings of the plane [3–5], the motivations have changed to the study of the geometrical properties of these patterns. The important role in the study of these tilings belongs to J. Conway and de Bruijn [6]. A tiling is a covering of the whole plane by shifts of a finite number of polygons, which don't overlap, and there are no holes. The tiling is not periodic (there exists no shift that does not change the tiling). But it is quasiperiodic. It means that any finite part of the tiling appears infinitely many times in the whole tiling. Some of the tilings have additional symmetry. Penrose tiling has the axis of the fifth order, which is forbidden for the periodic tiling. Another kind of symmetry is deflation. The initial polygons can be cut into pieces, these pieces can be sewn in such way that the new tiling is similar to the initial one. Below we shall discuss the tiling of the plane by two rhombuses, see Fig. 1. The deflation for these rhombuses is depicted in Fig. 2. The inverse transformation is called inflation.

R. Ammann and Mackay [7] proposed a three dimensional generalisation of Penrose nonperiodic tiling. The three dimensional tiling has icosahedral symmetry [8] which is incompatible with a periodic lattice. The experimental discovery of icosahedral symmetry (a rapidly quenched alloy of  $Al_6Mn$ ) [9] played an important role. Its crystalline structure was explained by the Penrose nonperiodic tiling [10–12]. (It was the first example of a quasicrystal.) This discovery has