Commun. Math. Phys. 109, 681–689 (1987)

Communications in
Mathematical
Physics

Invariants for Smooth Conjugacy of Hyperbolic Dynamical Systems. I

J. M. Marco and R. Moriyón*

Department of Mathematics, Universidad Autónoma de Madrid, E-28049 Madrid, Spain

Abstract. We prove that the Lyapunov exponents of periodic orbits are a total family of invariants for C^∞ conjugation of families of diffeomorphisms to a two-dimensional toral automorphism. In case of families of canonical diffeomorphisms, the corresponding conjugating diffeomorphisms are also canonical.

During the last few years there has been a considerable advance in perturbation theory of Anosov hamiltonian systems (see [GK, CEG, LMM]). In this paper we start the study of the corresponding C^∞ conjugation problem for general (not necessarily hamiltonian) systems. We prove in some particular situations that the simplest conjugation invariants, the Lyapunov exponents of periodic orbits, are a total family of invariants for C^∞ conjugation.

We also show that Lyapunov exponents are relevant to perturbation theory of hamiltonian Anosov systems. We prove that for hamiltonian systems of the class under consideration, the constancy of the Lyapunov exponents implies the constancy of the action invariants, which are known to be a complete family of invariants for canonical conjugation. This in turn implies that the Lyapunov exponents are (in our situation) a complete family of invariants for locally hamiltonian, as well as for hamiltonian, conjugation. We find this interesting since it is shown in [CEG] [see the comments following (1.9) and the example in Appendix E] that Lyapunov exponents are not a complete family of invariants for conjugation of locally hamiltonian perturbations of geodesic flows with negative curvature in space dimension 2 under globally canonical maps, and no positive result in this direction was known up to now. It is tempting to conjecture that our results can be extended to cover perturbations of suitable flows, at least in dimension 3.

We work in the context of one-parameter families of diffeomorphisms of the two-dimensional torus $T^2=\mathbb{R}^2/\mathbb{Z}^2$. Such a one-parameter family f_ε, $a<\varepsilon<b$, will

* Supported by CAICYT grant 397/84