Commun. Math. Phys. 109, 99–108 (1987)

Communications in
Mathematical
Physics

Calabi-Yau Manifolds as Complete Intersections in Products of Complex Projective Spaces

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Abstract. We consider constructions of manifolds with $SU(3)$ holonomy as embedded in products of complex projective spaces by imposing certain homogeneous holomorphic constraints. We prove that every such construction leads to one deformation class of manifolds with $SU(3)$ holonomy. For a subset of these manifolds we prove simple connectedness, address the problem of calculating the second Betti number and explicitly calculate it for a class of constructions. This establishes a very wide class of manifolds with $SU(3)$ holonomy, that can give rise to yet many more constructions via dividing out the action of suitably chosen discrete groups. Some of the examples studied may yield phenomenologically acceptable models.

1. Introduction

The existence of Ricci-flat Kähler manifolds was conjectured almost thirty years ago [1], but was proven only twenty years after that [2]. Manifolds of this type (generally called Calabi-Yau) of complex dimension n have $SU(n)$ holonomy and one covariantly constant everywhere non-vanishing and non-zero holomorphic n-form.

Owing to these features, Calabi-Yau manifolds of complex dimension 3 seem to have an immense impact on the analysis [3] of the phenomenology of superstring theories [4], hopefully leading to a phenomenologically acceptable model of unification of the known basic interactions and matter. Since the invariants of the complex structure of Calabi-Yau manifolds link tightly to the parameters of the physical models [3, 5], explicit constructions and computations of the corresponding invariants are necessary in this approach in order to make contact between the superstring theories and the real world of experiments.

In the usual analysis of the phenomenology [3], one requires a manifold of Euler character ± 6, multiply connected, with a preferrably big discrete structure group. The requirements on the set of discrete symmetries of the manifold as well as its structure group are rather model dependent though. The second Betti