

# Solutions of Hartree-Fock Equations for Coulomb Systems

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**Abstract.** This paper deals with the existence of multiple solutions of Hartree-Fock equations for Coulomb systems and related equations such as the Thomas-Fermi-Dirac-Von Weizäcker equation.

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## I. Introduction

We want to present here various existence results of multiple solutions of Hartree and Hartree-Fock equations for Coulomb systems. More precisely, we consider the

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