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The Essentially Commutative Dilations of Dynamical Semigroups on M_n

Burkhard Kümmerer[1]* and Hans Maassen[2]**

[1] Mathematisches Institut, Universität Tübingen, Auf der Morgenstelle 10, D-7400 Tübingen, Federal Republic of Germany

[2] Instituut voor Theoretische Fysica, KU Nijmegen, Toernooiveld, NL-6 525 ED Nijmegen, Netherlands

Abstract. For identity and trace preserving one-parameter semigroups $\{T_t\}_{t\geqq 0}$ on the $n \times n$-matrices M_n we obtain a complete description of their "essentially commutative" dilations, i.e., dilations, which can be constructed on a tensor product of M_n by a commutative W^*-algebra.

We show that the existence of an essentially commutative dilation for T_t is equivalent to the existence of a convolution semigroup of probability measures ρ_t on the group $\mathrm{Aut}(M_n)$ of automorphisms on M_n such that $T_t = \int_{\mathrm{Aut}(M_n)} \alpha d\rho_t(\alpha)$, and this condition is then characterised in terms of the generator of T_t. There is a one-to-one correspondence between essentially commutative Markov dilations, weak*-continuous convolution semigroups of probability measures and certain forms of the generator of T_t. In particular, certain dynamical semigroups which do not satisfy the detailed balance condition are shown to admit a dilation. This provides the first example of a dilation for such a semigroup.

Introduction

Dilations of semigroups of completely positive operators on W^*-algebras can be studied under two different points of view: If the W^*-algebras are commutative then the semigroup of (completely) positive operators can be interpreted as a semigroup of transition operators, and its Markov dilation turns out to be the corresponding Markov process. Therefore, from a probability theoretic point of view, a Markov dilation is a non-commutative Markov process or a quantum Markov process.

On the other hand a semigroup of completely positive operators on a W^*-algebra can be interpreted as an operator algebraic description of an irreversibly behaving physical system. In this frame a dilation is a larger reversibly evolving system from which the irreversible system is recovered by coarse graining.

A fundamental problem in non-commutative probability theory is to find all

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