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One Dimensional Spin Glasses with Potential Decay $1/r^{1+\varepsilon}$. Absence of Phase Transitions and Cluster Properties

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Abstract. One-dimensional Ising spin systems interacting via a two-body random potential are considered; a decay with the distance like $1/r^{1+\varepsilon}$ is assumed.

We consider only boundary conditions independent of the random realization of the interactions and prove uniqueness and cluster properties of Gibbs states with probability one.

1. Introduction

Spin glasses are at present one of the major areas of interest in Statistical Mechanics. Only few problems have so far been solved in a rigorous way. In particular the existence and the nature of phase transitions are still open problems even in the Sherrington-Kirkpatrick mean field theory, for which however a very precise heuristic theory exists (see [10]).

As far as rigorous results are concerned, we mention the proof of the existence of thermodynamics for interactions decaying like $r^{-\alpha d}$ with $\alpha > 1/2$ in d dimensions [4, 7, 9]. Khanin [8] proved the uniqueness of Gibbs distribution in one dimension for interactions decaying like $r^{-\alpha}$ with $\alpha > 3/2$. Cassandro et al. [2] proved under the same conditions the infinite differentiability of thermodynamic functions.

The one-dimensional case with $1 < \alpha \leqq 3/2$ appears qualitatively different from the former case, since here it is not true that the supremum of the interaction among two contiguous half-lines over all spin configurations is finite with probability one. This case has been considered in [5], where the authors deal with the problem of absence of symmetry breaking. They show essentially that the interaction among two contiguous half-lines is bounded if one excludes a subset of "bad" spin configurations of zero Gibbs measure. The situation is reminiscent of superstable unbounded spins (see [1]), but here the set of bad configurations depends on the random realization of the interaction.

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