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A Mathematical Reformulation of Derrida's REM and GREM

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Abstract. The large system limit of the Random Energy Model (REM) and generalized Random Energy Model (GREM) of Derrida is investigated, and found to be universal. This permits systematic calculations of relevance in particular to Parisi's solution of the Sherrington-Kirkpatrick spin-glass model.

Introduction

B. Derrida has recently introduced two statistical models called respectively Random Energy Model (REM) [2, 3] and Generalized Random Energy Model (GREM) [4, 5]. These models are particularly interesting because they describe the thermodynamic behavior of the Sherrington–Kirkpatrick (SK) model [14, 8] expected on the basis of Parisi's Ansatz. For a discussion of the SK model in the light of Parisi's Ansatz [12, 13], we refer to Mézard et al. [9, 11], and references quoted there. For the connection with the REM and GREM, see Mézard, Parisi and Virasoro [10], Derrida and Toulouse [6] and, most clearly, de Dominicis and Hilhorst [7].

In Derrida's formulation of the REM and GREM, certain limits are implicit ($N \to \infty$, and for the GREM, number of levels of the hierarchy $\to \infty$). The purpose of the present paper is to give a mathematical reformulation where the appropriate limits have already been taken. Our approach has the advantage of showing that no hidden difficulties lurk behind these limits. It also permits an easier discussion of certain problems, as we shall see below.

We shall proceed dogmatically by defining certain spaces and probability measures. The connection with Derrida's definitions should then be rather clear, and is discussed only briefly.

1. Poisson Distributions

The usual Poisson distribution describes (infinite) configurations of points on the line $\mathbb{R}$, with given density φ, and such that any two disjoint intervals of $\mathbb{R}$ behave independently. This setup can be variously generalized, we shall use an extension where $\mathbb{R}$ is replaced by a nonempty open set $\mathcal{O} \subset \mathbb{R}^\nu$, and the density by a continuous function $\varphi \geqslant 0$ on $\mathcal{O}$. A configuration X of points in $\mathcal{O}$ will be represented by an