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Scattering Off of an Instanton

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Abstract. We consider the scattering of a classical colored particle off an instanton. That is, we investigate Wong's equations (or equivalently, the Kaluza-Klein geodesic equations) for a color $SU(2)$ particle under the influence of a Euclidean instanton. We solve the equations in the limit in which the instanton becomes singular. Our main result is that particles with head-on trajectories scatter off the instanton with a scattering angle of $\pi/3$. This angle is independent of the magnitude of the color charge and velocity of the particle as long as both are nonzero. The plane in which the scattering takes place is determined by the particle's initial position and color charge. We also solve for the geodesics for the corresponding (singular) Kaluza-Klein metric on S^7.

1. Wong's Equations

Some History

Wong (1970) introduced equations of motion for a classical colored spinless particle under the influence of an external Yang-Mills potential A. The equations reduce to the Lorentz equations in the abelian case. See Arodz (1982) or Balachandran et al. (1983) for further discussion.

Kaluza-Klein (1921) proposed an alternative framework in which to describe such a particle. In their framework the particle travels in a geodesic relative to a certain metric on a principal bundle over space-time. Kerner (1968) generalized Kaluza-Klein's idea from the abelian to the non-abelian case. The Wong and the Kaluza-Klein formulations were symplecticized by Sternberg (1978) and Weinstein (1978) respectively. See Sniatycki (1979) or Montgomery (1984) for a further discussion of symplectic aspects.

Wong's Equations over a General Manifold

Let X be a Riemannian manifold with metric g. We think of X as the space on which our classical colored particle travels. Let G be a compact Lie group and $\mathfrak{g}$ its Lie algebra. We think of the dual $\mathfrak{g}^*$ of the Lie algebra as the space of "color charges," or internal degrees of freedom, for our colored particle. Fix a bi-invariant