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A Phase Cell Approach to Yang-Mills Theory

I. Modes, Lattice-Continuum Duality*

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Abstract. For the abelian Yang-Mills theory, a one-to-one correspondence is established between continuum gauge potentials and compatible lattice configurations on an infinite sequence of finer and finer lattices. The compatibility is given by a block spin transformation determining the configuration on a lattice in terms of the configuration on any finer lattice. Thus the configuration on any single lattice is not an "approximation" to the continuum field, but rather a subset of the variables describing the field.

It is proven that the Wilson actions on the lattices monotonically increase to the continuum action as one passes to finer and finer lattices. Configurations that minimize the continuum action, subject to having the variables fixed on some lattice, are studied.

0. Introduction

We consider an infinite sequence of finer and finer lattices. To each bond of each lattice there is assigned a group element, in the additive group of real numbers. There is a compatibility requirement to these assignments; the assignments to any one of the lattices are determined in terms of the assignments to any finer lattice, by an averaging procedure due to Balaban. Given a continuously differentiable gauge potential, $A_\mu(x)$, one can define compatible assignments to the lattices, as above, such that, in a suitable sense, the lattice "fields" approach $A_\mu(x)$ as one passes to finer and finer lattices. The Wilson actions likewise approach the continuum action $\frac{1}{2}\int (dA)^2$. A particularly useful feature of the above duality between lattice fields and continuum fields is the following: let p be a plaquette in any of the lattices, then the group assignment to p, $A_{\partial p}$, is given in terms of an integral with the continuum field

$$A_{\partial p} = \int \chi_p(x) \cdot \mathbf{A}(x), \tag{0.1}$$

where $\chi_p(x)$ is a function associated to p.

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