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Synchronisation of Canonical Measures for Hyperbolic Attractors

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Abstract. Under suitable conditions it is shown how to change the velocity of a C^2 Axiom A attractor so that the Sinai-Ruelle-Bowen measure coincides with the measure of maximal entropy. These measures are obtained as limits of certain closed orbital measures.

There are two invariant (probability) measures which come to the fore in the study of C^2 hyperbolic attractors (and no doubt in the study of dynamical systems satisfying less stringent hyperbolic conditions). They are (i) the measure of maximal entropy and (ii) the Sinai-Ruelle-Bowen (S.R.B.) measure, which, in case the flow preserves a smooth measure is this measure. There are many examples where these two measures differ and many where they coincide.

The aim of this note is to show that it is possible, under suitable conditions, to change the velocity so that the corresponding measures for the new flow coincide. This change of velocity is brought about by the multiplication of speeds by a function which is essentially unique. In other words the "reason" for the two canonical measures differing appears to be that the system is running at the wrong speed and that there is a canonical speed (up to constant scalar factors) for the 1 dimensional foliation of the flow.

A secondary purpose of this note is to describe the S.R.B. measure from an *internal* point of view as opposed to the usual external definition in terms of the behaviour of (Lebesgue) almost all points. Our description is in terms of weighted orbital measures. Unweighted orbital measures converge weakly to the measure of maximal entropy according to a theorem of Bowen [4] whereas weighted orbital measures converge to the S.R.B. measure (for appropriate weights, of course). The latter fact has been observed and conjectured for certain systems by Hannay and Ozorio De Almeida [6].

In Sect. 5 we discuss, briefly, the Ruelle zeta function, which captures periodic orbital data and is an invariant of topological conjugacy. In contrast to this function we introduce (for C^2 hyperbolic attractors) a different zeta function which is invariant under C^1 velocity changes (but not topological conjugacy). Under appropriate conditions the synchronisation mentioned above yields a flow for