Commun. Math. Phys. 106, 201–210 (1986)

Communications in Mathematical Physics

Spinor Two-Point Functions in Maximally Symmetric Spaces

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Abstract. The two-point function for spinors on maximally symmetric four-dimensional spaces is obtained in terms of intrinsic geometric objects. In the massless case, Weyl spinors in anti de Sitter space can not satisfy boundary conditions appropriate to the supersymmetric models. This is because these boundary conditions break chiral symmetry, which is proven by showing that the "order parameter" $\langle \bar{\psi}\psi \rangle$ for a massless Dirac spinor is nonzero. We also give a coordinate-independent formula for the bispinor $S(x)\bar{S}(x')$ introduced by Breitenlohner and Freedman [1], and establish the precise connection between our results and those of Burges, Davis, Freedman and Gibbons [2].

1. Introduction

Maximally symmetric spacetimes provide an interesting background for studying quantum field theory in curved space. They also have nice applications: de Sitter space (DS) appears in "inflationary" models of the early universe [3], and anti de Sitter space (ADS) as the classical ground state of gauged supergravity models [1].

In this paper we extend the coordinate independent construction of two-point functions for bosons [4] to the fermionic case. The method employs only geometric quantities intrinsic to the manifold, such as the propagator of parallel transport. By exploiting the maximal symmetry of the spacetime we therefore obtain very simple expressions.

We use notation in which spinors ξ^A (conjugate spinors $\bar{\xi}^{A'}$) have undotted (dotted) capital latin indices. This notation is explained in [5], and restricts the applicability of this work to four dimensions.

In Sect. two we introduce the parallel propagator for spinors and calculate its covariant derivative. Section three uses this result to find the massive two-point functions for DS and ADS. Section four treats the massless limits, and shows why chiral symmetry must be broken in ADS. In Sect. five we obtain a simple formula for the Killing bispinor $S(x)\bar{S}(x')$ introduced in [1, 2], and use it to establish the precise equivalence between our results and theirs.

The conclusion—that chiral symmetry is broken by supersymmetric boundary