Commun. Math. Phys. 105, 645–668 (1986)

Communications in Mathematical Physics

Supersymmetry in a Space with Auxiliary Dimensions

A. A. Rosly and A. S. Schwarz

Institute of Theoretical and Experimental Physics, SU-117259 Moscow, USSR

Abstract. The purpose of this paper is to clarify the geometrical constructions leading to unconstrained superfields in extended supersymmetry.

1. Introduction

Recently great progress was made in constructing off-shell superfield theories in the case of extended supersymmetry. Galperin, Ivanov, Kalitzin, Ogievetsky, and Sokatchev succeeded in finding unconstrained superfield formulations for supersymmetric $N=2$ matter, supergravity, and $N=2$ and 3 Yang-Mills theories [1, 2]. The crucial point that allowed them to overcome the $N=3$ barrier (for maximum spin one, and $N=2$ barrier for maximum spin one half) was the use of special superspaces with auxiliary variables. These superspaces emerged previously in an analog of Ward's twistor construction [3] which was suggested in [4] for the purpose of solving $N=2$ and $N=3$ superspace constraints. As a matter of fact, the description of $N=2$, 3 unconstrained superfields and their relations with superfields in the usual Minkowski superspace can be made very close to the Ward construction (as well as to some other twistor constructions based on Ward's idea; see [5–7]). The purpose of the present paper is to clarify the geometrical origin of new unconstrained superfields introduced by Galperin et al. and to show how to come naturally to the results of [1, 2] starting from an analog of Ward transformation considered in [4]. This study has led us also to the construction of an unconstrained off-shell formulation for the general $N=2$ supersymmetric hyper-Kähler sigma model, which will be described elsewhere. (Interestingly enough, the treatment of the hyper-Kähler manifold proper to $N=2$ supersymmetry demonstrates once again a close analogy with twistor constructions – this time with a description of gravitational instantons due to Penrose [5].)

The key idea of introducing auxiliary dimensions borrowed from twistor constructions is based on the following simple observation. It turns out that if the extra dimensions added to some space form a compact complex manifold, then there is no actual increase of dimension for those fields, which are holomorphic with respect to these extra variables. This follows from the statement that any