

A Direct Variational Approach to Skyrme's Model for Meson Fields

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Abstract. The solutions of Skyrme's variational problem describe the structure of mesons in a field of weak energy. The problem consists in minimizing the corresponding energy among the functions from $\mathbb{R}^3$ to S^3 which have a fixed "degree" without making any symmetry assumptions. We prove the existence of minima and study their properties.

Introduction

Let ϕ be a function from $\mathbb{R}^3$ to $\mathbb{R}^4$ such that $\phi(\mathbb{R}^3)$ is contained in the unit sphere of $\mathbb{R}^4$. If we write

$$|\nabla\phi|^2 \equiv \sum_{i=1}^4 |\nabla\phi_i|^2 \quad \text{and} \quad |A(\phi)|^2 = \sum_{\alpha,\beta=1}^3 \left| \frac{\partial\phi}{\partial x_\alpha} \wedge \frac{\partial\phi}{\partial x_\beta} \right|^2$$

(by $a \wedge b$ we denote the alternating exterior product of $a, b \in \mathbb{R}^4$), we will define the energy associated with the fields $\{\phi_i\}$ by:

$$\mathcal{E}(\phi) = \frac{\gamma}{4\pi^2} \int_{\mathbb{R}^3} (\kappa^2 |\nabla\phi|^2 + |A(\phi)|^2) dx,$$

where γ and κ are two positive physical constants.

Physical Motivation. T. H. R. Skyrme introduced in [S3] the problem of looking for critical points of the functional $\mathcal{E}$ in an attempt to find a model for the static configurations of a field of mesons.

Recent works seem to indicate that in the large N -limit some gauge field theories (QCD) are equivalent to an effective field theory of mesons (see [W]). Unfortunately, little is known about the large N limit. Some authors have suggested the study of the case $N = 3$, hoping that this case will be very close to the large N -limit (see G. t'Hooft [t'H 1, 2]).

Another possibility is to attempt simplified field theories as was proposed by Skyrme. Skyrme's idea consisted in adding to the nonlinear σ -model