Commun. Math. Phys. 105, 385–390 (1986)

Communications in Mathematical Physics

Crystals and Quasicrystals: A Continuum Model*

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Abstract. We construct the first model of particles in the plane with completely symmetric, short range, two body interactions which has quasiperiodic, but no periodic, ground states.

1. Introduction

It is a problem of fundamental importance [1–5] to determine why, at low temperature, real matter strongly tends to have crystalline symmetry at the molecular level. The recent discovery [6] of quasicrystalline matter gives further impetus to understand to what extent, and especially by what mechanism, low temperature prescribes the symmetry of configurations of many interacting particles.

Using a grand canonical ensemble for several particle species, the low temperature distribution is concentrated on configurations with low value of $(e - \Sigma_j m_j d_j)$, where e is the energy density and m_j (respectively d_j) is the chemical potential (respectively particle density) of the jth species. The symmetry of these configurations is the matter at issue.

Results on this "crystal problem" [7–23] have concentrated on classical mechanical models, mostly lattice gas models. The problem is essentially solved for one dimension, both for lattice gas [19] and continuum [20] models. Among two dimensional models a class with highly symmetric interactions is known [15, 21, 23] to have periodic ground states. In contrast, recent results [21, 23] have exhibited lattice gas models with *no* periodic ground states.

Using these recent lattice gas models we will construct the first model of particles in the plane (i.e. a continuous model) with completely symmetric, short range, two body interactions which has quasiperiodic, but *no* periodic, ground states. This will require that we also significantly enlarge the class of continuous models known to have periodic ground states.

* Supported in part by NSF Grant No. DMS-8501911