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Translation Invariant Gibbs States in the q-State Potts Model

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Abstract. We describe the set of all translation invariant Gibbs states in the q-state Potts model for the case of q large enough and the other parameters to be arbitrary.

Introduction

The aim of this note is to describe the set of all translation invariant Gibbs states in the q-state Potts model. We consider only the case of q large enough, assuming the other parameters of the model, i.e. the temperature and the space dimension $\nu \geqq 2$, to be arbitrary. Let $\mathbb{Z}^\nu$ be ν-dimentional lattice, $\nu \geqq 2$. The distance between any two points $x, y \in \mathbb{Z}^\nu$, $x=(x_1, \ldots, x_\nu)$, $y=(y_1, \ldots, y_\nu)$, is defined as $d(x, y)=\sum_{i=1}^{\nu} |x_i - y_i|$. We assume that the spin $\varphi(x)$, $x \in \mathbb{Z}^\nu$, in the model under consideration takes values in the finite set $Q=\{1, \ldots, q\}$, and the formal Hamiltonian is written as follows:

$$H=-\sum_{\langle x, y\rangle} \delta_{\varphi(x), \varphi(y)}, \qquad \varphi(x), \varphi(y) \in Q, \tag{1}$$

where the sum is taken over all the pairs of nearest neighbors x, y on the lattice and δ is the Kronecker symbol. By $\mathrm{g}(\beta, q)$ [respectively by $\mathrm{g}^{(\mathrm{inv})}(\beta, q)$] is denoted the class of all (respectively of all translation invariant) Gibbs states with β parameter and the Hamiltonian (1).

By using reflection positivity Kotecky and Shlosman [1] have proved the coexistence of $q+1$ phases at some $\beta_c(q)$ (critical inverse temperature) for q large enough. Another approach to the solution of this problem, based on the contour technique, was offered by E. Dinaburg and Ya. Sinai [2] and independently by Bricmont et al. [3]. Everywhere below we mean that the value of $\beta_c(q)$ is defined namely as in [2], although the next theorem shows that $\beta_c(q)$ is to be unique.

Now we formulate the main result of this paper.

Theorem. *For any $\nu \geqq 2$, $q_0(\nu)$ may be found so that for all $q > q_0(\nu)$ the following statement is true. There exists such a value $\beta = \beta_c(q)$ of inverse temperature, that*