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Asymptotic Completeness for a Quantum Particle in a Markovian Short Range Potential

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Abstract. Absence of bound states and asymptotic completeness are proven for a quantum particle in a time dependent random (Markovian) short range potential. Systems with confining potentials are also considered and unboundedness of the energy in time is shown.

1. Introduction and Results

In a previous paper ([1]) we started studying the quantum dynamics generated by random time dependent Hamiltonians of the form

$$H(t) = H_0 + V(\xi(t)), \tag{1.1}$$

where H_0 is a self adjoint operator on some Hilbert space $\mathscr{H}$ (typically $\mathscr{H} = L^2(\mathbb{R}^\nu)$ or $l^2(\mathbb{Z}^\nu)$ with $H_0 = -\Delta$), $\{\xi(t)|t\in\mathbb{R}\} = \underset{\sim}{\xi}$ a path of a stationary Markov process on some state space E with a unique invariant measure μ and $V(\cdot)$ a function on E with values in the self adjoint operators on $\mathscr{H}$.

In this paper we continue the analysis of such systems. The first and main part of our work is devoted to the case $\mathscr{H} = L^2(\mathbb{R}^\nu)$ for $\nu \geqq 3$ and $H_0 = -\Delta$, $V(\xi)$ multiplication by a short range potential $V(\xi, x)$ (i.e. sufficiently rapidly decaying at spatial infinity). From [1] we learn modulo some non-triviality condition assuring (1.1) to be "sufficiently time dependent" that such a system leaves any bounded region of its phase space in time mean (this is the "RAGE-theorem" 4.2 in [1]); however we don't know how. It may tend to spatial infinity, or have unbounded kinetic energy, or both. We only know states with bounded energy to approach spatial infinity like a free particle (from Corollary 4.4 in dimensions $\nu \geqq 5$). We prove this to be the right behaviour in general. More technically we show the dynamics generated by (1.1) to be asymptotically complete (with respect to the free one). Let us formulate our result as a

Theorem. *Let $H_0 = -\Delta$ be the ordinary kinetic energy on $L^2(\mathbb{R}^\nu)$ with $\nu \geqq 3$. Further assume the short range potential $V(\xi, x)$ and the process $\xi(\cdot)$ to satisfy the conditions 2.1–2.5 of Sect. 2. Then if $U(\underset{\sim}{\xi}|t, s)$ denotes the unitary propagator associated to (1.1),*