Commun. Math. Phys. 105, 133–152 (1986)

Communications in
Mathematical
Physics

Bulk Transport Properties and Exponent Inequalities for Random Resistor and Flow Networks

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Abstract. The properties of random resistor and flow networks are studied as a function of the density, p, of bonds which permit transport. It is shown that percolation is sufficient for bulk transport, in the sense that the conductivity and flow capacity are bounded away from zero whenever p exceeds an appropriately defined percolation threshold. Relations between the transport coefficients and quantities in ordinary percolation are also derived. Assuming critical scaling, these relations imply upper and lower bounds on the conductivity and flow exponents in terms of percolation exponents. The conductivity exponent upper bound so derived saturates in mean field theory.

1. Introduction and Summary of Results

The problem of bulk transport in random resistor networks has attracted a great deal of attention for almost two decades [1–22]. The simplest such network is one in which the bonds of a regular lattice are taken to be occupied independently by unit conductors with probability p, and vacant (i.e., of infinite resistance) with probability $1-p$. Interest in this model is a consequence of the suggestion, due to Ziman [1], Eggarter and Cohen [2, 3] and others, that it provides a good description of the conductivity properties of (binary) inhomogeneous conductors when the disorder occurs on an intermediate scale. In particular, it was argued that the resistor network is capable of bulk conductivity if and only if p exceeds the percolation threshold, p_c, of the lattice, i. e. that the resistive transition occurs at the percolation threshold.

Given the arguments for coincidence of these transition points, it was initially expected [1] that the bulk conductivity, $\sigma(p)$, should be proportional to the percolation probability, $P_\infty(p)$. However, this assumption was soon called into question by the table-top experiment of Last and Thouless [4]. If, following

[1] Research supported by the NSF under Grant No. DMR-8314625
[2] Research supported by the DOE under Grant No. DE-AC02-83ER13044