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One-Dimensional Classical Massive Particle in the Ideal Gas

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Abstract. The motion of a one-dimensional massive particle under the action of collisions with points of the ideal gas is considered. It is shown that the normed displacement of the massive particle is represented asymptotically as the difference of random variables having limit Gauss distribution. Estimations of the diffusion coefficient not depending on the mass are found.

1. Description of the Model and Formulation of the Results

The probabilistic model of the Brownian particle was constructed more than fifty years ago. One of the first references is the classical paper of Wiener [11]. Since that time the Wiener measure became a subject of many deep investigations. Now it is a beautiful chapter of the theory of random processes which can be found practically in all text-books on the subject (see [4, 6]). It is surprising enough that a general mechanical model of the Brownian particle was not constructed so far. Only several particular cases were considered in [7, 8]. Moreover, it was discovered recently in direct experiments (see [5]) that for the Brownian particle the correlation functions for the velocity decay only as a power of time which shows that the representation of the displacement of the Brownian particle as a sum of independent or weakly dependent random variables is a crude approximation. It is worthwhile to mention also that after the discovery of Alder and Wainright (see [2]) such decay of correlations is typical for many problems of non-equilibrium statistical mechanics (see [1]).

The goal of this paper is to present several rigorous results concerning the asymptotic behaviour of a massive particle (m.p.) of mass M moving in one direction under the action of elastic collisions with particles of equal masses whose masses are taken to be equal to 1. It is assumed that the particles do not interact and their distribution is the equilibrium distribution of the ideal gas with density ϱ and inverse temperature β. The coordinate and velocity of the m.p. are denoted by $q_0, v_0, x_0=(q_0, v_0)$. A collection of equal particles is denoted by $X=\{x\}$, $x=(q, v)$, and $Y=(x_0, X)$. The phase space of all possible Y is denoted by Ω. For any subset