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Stability of Coulomb Systems with Magnetic Fields

II. The Many-Electron Atom and the One-Electron Molecule

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Abstract. The analysis of the ground state energy of Coulomb systems interacting with magnetic fields, begun in Part I, is extended here to two cases. Case A: The many electron atom; Case B: One electron with arbitrarily many nuclei. As in Part I we prove that stability occurs if $z\alpha^{12/7} < \text{const}$ (in case A) and $z\alpha^2 < \text{const}$ (in case B), ($z|e|$ = nuclear charge, α = fine structure constant), but a new feature enters in case B. There one *also* requires $\alpha < \text{const}$, regardless of the value of z.

I. Introduction

In the first paper in this series [1] the question of the stability of atoms and molecules in the presence of magnetic fields was raised, and it was answered in the case of the one-electron atom of arbitrary nuclear charge $z|e|$. In the present paper the stability question will be answered in two other cases:

(A) The many electron atom,

(B) The one-electron molecule.

Unfortunately, the stability of the many-electron, many-nucleus system is still an open question.

The reader is referred to the introduction in [1] for the motivation and physical interpretation of this problem. The mathematical essence of the problem is that we want to decide whether or not the energy functional

$$\mathscr{E}(\psi, A, \underline{R}, \underline{z}) \equiv \sum_{j=1}^{N} \int |\sigma_j \cdot (p_j - A(x_j))\psi|^2\, dx + \varepsilon \int B(x)^2\, dx + (\psi, V(X, \underline{R}, \underline{z})\psi) \tag{1.1}$$

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