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An Extension of Kotani's Theorem to Random Generalized Sturm–Liouville Operators

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Abstract. We consider the random operator: $-d/m_\omega(dx)d^+/dx + q_\omega(x)$, where $m_\omega(dx)$ and $q_\omega(x)$ are a stationary ergodic random measure and a random function respectively. To this general case, we extend Kotani's theorem which asserts that the absolutely continuous spectrum is completely determined by the Ljapounov indices. Our framework includes the case of stochastic Jacobi matrices treated by Simon.

1. Introduction

In [9] Kotani treated the one-dimensional Schrödinger operator with a stationary, ergodic, bounded potential:

$$-\frac{d^2}{dx^2} + q_\omega(x),$$

and proved that with probability one its absolutely continuous spectrum coincides, up to a set of Lebesgue measure zero, with the totality of real numbers for which Ljapounov indices vanish. Simon [13] proved that this theorem holds also for stochastic Jacobi matrices of the following type:

$$H_\omega u(n) = -u(n+1) - u(n-1) + V_\omega(n)u(n).$$

Our aim here is to extend Kotani's theory, which includes the abovementioned theorem, to the random generalized Sturm–Liouville operator:

$$L_\omega u(x) = -\frac{d}{m_\omega(dx)}\frac{d^+u}{dx} + q_\omega(x)u(x), \tag{1.1}$$

where $m_\omega(dx)$ is a stationary ergodic random Radon measure on $\mathbb{R}^1$; $d^+u/dx = u^+(x)$ is the right-derivative, $d/m_\omega(dx)$ is the Radon–Nikodym derivative, and $q_\omega(x)$ is a bounded random function defined for $x \in \mathrm{Supp}(m_\omega)$. Here $\mathrm{Supp}(m_\omega)$ is the totality of x such that $m_\omega((x-\varepsilon, x+\varepsilon)) > 0$ for any $\varepsilon > 0$. It will be shown that most of the results in [9], including the methods of proofs, hold also for this random operator.

This class of operators contains Schrödinger operators with random potentials