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Debye-Hückel Limit of Quantum Coulomb Systems

I. Pressure and Diagonal Reduced Density Matrices

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Abstract. In this paper, we consider charge symmetric quantum Coulomb systems with Boltzmann statistics. We prove that the theory of screening of Debye and Hückel is a combined classical and mean field limit of these quantum Coulomb systems.

Introduction

Quantum Coulomb systems are known to be stable: the thermodynamic functions of these systems, as well as their convexity properties have been obtained from the principles of statistical mechanics by Lieb and Lebowitz [1]. The microscopic properties of these systems are however far from being understood. For instance, nothing is known about the clustering properties of their reduced density matrices (R.D.M.).

In this paper, we show that the classical theory of screening of Debye and Hückel (see for instance [2, p. 275]) is an exact classical and mean field limit of quantum Coulomb systems. For technical reasons, we have to impose restrictions on the class of systems we consider. We restrict ourself to charge symmetric systems in the Grand Canonical Ensemble; for a two component system (which is the case we consider for simplicity), this means that the activities $\underset{\sim}{z}$, masses m, and absolute value of charge e of both species have to be the same. Moreover, we only deal with quantum systems with Boltzmann statistics (we shall have to add short range forces to insure stability). For such models Fröhlich and Park have been able to prove the existence of the thermodynamic limit of the reduced density matrices [3]. These systems are described by three parameters: $\beta = e^2(kT)^{-1}$, $\alpha = \hbar^2(me^2)^{-1}$, $z = \underset{\sim}{z}(\alpha\beta 2\pi)^{-3/2}$. From these 3 parameters only 2 are independent. Indeed, because of the scaling properties of the Coulomb potential, the system described by the parameters (β, α, z) is equivalent to the one with parameters $(\beta/\ell, \alpha/\ell, z/\ell^3)$, where ℓ is any non-zero positive number which represents a change of length-scale.

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