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# Unitary Representations of the Virasoro and Super-Virasoro Algebras

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**Abstract.** It is shown that a method previously given for constructing representations of the Virasoro algebra out of representations of affine Kac-Moody algebras yields the full discrete series of highest weight irreducible representations of the Virasoro algebra. The corresponding method for the super-Virasoro algebras (i.e. the Neveu-Schwarz and Ramond algebras) is described in detail and shown to yield the full discrete series of irreducible highest weight representations.

## 1. Introduction

In a recent letter [1] we described a method for constructing representations of the Virasoro algebra out of representations of affine Kac-Moody algebras. The Virasoro algebra occurs as the algebra of the conformal group in one dimension, or, in the form of two commuting copies, in two dimensions. Thus it is of importance in physical contexts where two-dimensional conformal invariance plays a crucial rôle, such as string theories or the behaviour at critical points of two-dimensional statistical systems [2, 3]. The Virasoro algebra is defined by the commutation relations

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m,-n}, \qquad m,n\in\mathbb{Z}, \tag{1.1}$$

where $c$ is a central element, i.e. $[L_n, c]=0$, so that $c$ is assigned a numerical value in any irreducible representation. In this paper we shall be concerned with unitary representations of this algebra, that is representations satisfying the hermiticity conditions,

$$L_n^\dagger = L_{-n}, \tag{1.2}$$

and, more particularly, highest weight representations, that is ones in which all the

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