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A Renormalizable Field Theory: The Massive Gross-Neveu Model in Two Dimensions

Joel Feldman[1], J. Magnen[2], V. Rivasseau[2], and R. Sénéor[2]

[1] Mathematics Department, University of British Columbia, Vancouver, B.C., Canada V6T 1Y4
[2] Centre de Physique Théorique, Ecole Polytechnique, F-91128 Palaiseau Cedex, France

Abstract. The Euclidean massive Gross-Neveu model in two dimensions is just renormalizable and asymptotically free. Thanks to the Pauli principle, bare perturbation theory with an ultra-violet cut-off (and the correct ansatz for the bare mass) is convergent in a disk, whose radius corresponds by asymptotic freedom to a small finite renormalized coupling constant. Therefore, the theory can be fully constructed in a perturbative way. It satisfies the O.S. axioms and is the Borel sum of the renormalized perturbation expansion of the model

I. Introduction

The Gross-Neveu (or Mitter-Weisz) models in two dimensions (in short the GN_2 models) are among the simplest physical field theories which are asymptotically free, hence are among the most obvious "laboratories" to investigate the perturbative behavior of non-abelian gauge theories in four dimensions, and non-perturbative phenomena like spontaneous symmetry breaking [1, 2]. They are models of N-component fermions (with $N \geqq 2$), with a quartic interaction; therefore their graphs are topologically the same as those of the familiar Φ^4 bosonic theories.

Although the models discussed by Gross and Neveu were massless, and their paper [2] was mostly devoted to the discussion of chiral symmetry breaking and dynamical mass generation, the massive version (i.e., with non-zero *bare* mass) makes perfect sense. In contrast with the massless case these massive GN_2 theories, while still asymptotically free [1], should have a purely perturbative content (by this we mean that all the physics of the model could be extracted from ordinary perturbation theory). Therefore, they are the simplest candidates for a rigorous construction of a renormalizable field theory, and it is slightly surprising that they were not often presented as such by "constructive field theory." (Of course these models are only two-dimensional; we remark also that the case $N=1$, which corresponds to the massive Thirring model has been solved by means of the sine Gordon transformation [3].)