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On Purely Radiative Space-Times(*)

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Abstract. The existence of space-times representing pure gravitational radiation which comes in from infinity and interacts with itself is discussed. They are characterized as solutions of Einstein's vacuum field equations possessing a smooth structure at past null infinity which forms the "future null cone at past timelike infinity with complete generators." The "pure radiation problem" is analysed where "free initial data" for Einstein's field equations are prescribed on the null cone at past time-like infinity. It is demonstrated how the pure radiation problem can be formulated as a local initial value problem for the symmetric hyperbolic system of reduced conformal vacuum field equations. Its solutions are uniquely determined by the free data.

1. Introduction

This paper is concerned with space-times representing gravitational radiation, which falls in from infinity, interacts nonlinearly with itself, and eventually escapes to infinity again. It is clear that a sufficiently complete understanding of this process and of the global structure of the resulting field is of fundamental importance for classical as well as for quantum gravity. Due to the work of Pirani [20], Sachs [21, 22], Bondi et al [1], Newman and Penrose [12], Penrose [15–17], Geroch [10] and others, there exists a rigorous setting in which to study the situation described above, without having to take recourse to approximation procedures. One will expect that the field is a solution of Einstein's vacuum field equations which has a smooth structure at past null infinity. To make sure that the field is indeed built up only by incoming gravitational radiation, one will require that the null generators of past null infinity satisfy a certain completeness condition and that past timelike infinity is represented in the conformal completion by a point i^-. This point is required to be "regular" in the sense that the conformally rescaled "unphysical" space-time admits a smooth extension in which i^- is a regular point and such that the future null cone of i^- represents past null infinity for the physical

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