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On the Existence of the Real Time Evolution in Euclidean Lattice Gauge Theories

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Abstract. A simple argument is given which excludes the occurrence of zero eigenvalues of the transfer matrix in euclidean lattice gauge theories.

1. Introduction

If the transfer matrix T of a classical statistical mechanics is positive and has no zero eigenvalue, one can associate to this theory a quantum theory where the Hamiltonian H is defined by $H = -\ln T$. The positivity of T is implied by the reflection positivity of the statistical mechanics; euclidean lattice gauge theories have this property for the standard actions [1, 2]. The absence of zero eigenvalues of T in the thermodynamic limit, however, is an apparently hard problem which remained unsolved over several years [3].

In striking contrast to the difficulties which complicate a direct proof, the solution becomes simple if one formulates the problem in terms of the algebra of observables of the quantum theory. This is possible for finite gauge groups and has first been carried out in [4]. The argument follows:

The local transfer matrices T_Λ implement (non-*-) automorphisms of the algebra of local observables $\mathfrak{A}$,

$$\alpha_i^\Lambda(A) = T_\Lambda A T_\Lambda^{-1}. \tag{1.1}$$

In a theory with finite range interactions these automorphisms converge to a (non-*-) automorphism α_i of $\mathfrak{A}$, which may be interpreted as time translation by one unit in imaginary direction.

A ground state ω_0 is defined to be a state with

$$\begin{aligned} &\text{(i)} && \omega_0 \alpha_i = \omega_0, \\ &\text{(ii)} && 0 \leqq \omega_0(A^* \alpha_i(A)) \leqq \omega_0(A^* A), \quad A \in \mathfrak{A}, \end{aligned} \tag{1.2}$$

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