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A Note on the Covariant Anomaly as an Equivariant Momentum Mapping

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Abstract. We show that there is a natural gauge invariant presymplectic structure ω on the space $\mathscr{A}$ of all vector potentials. The covariant axial anomaly $\tilde{G}$ is found to be the essentially unique infinitesimally equivariant momentum mapping for the action of the group of gauge transformations on $(\mathscr{A}, \omega)$. The infinitesimal equivariance of $\tilde{G}$ is shown to be equivalent to the Wess-Zumino consistency condition for the consistent axial anomaly G. We also show that the X operator of Bardeen and Zumino, which relates G and $\tilde{G}$, corresponds to the one-form (on $\mathscr{A}$) of the presymplectic structure ω.

Introduction

The mathematical structure of the consistent axial anomaly G can be studied from several viewpoints. For example, one can use differential geometric and algebraic techniques on spacetime, as in Zumino [16] and Zumino et al. [17]; or one can use differential geometry and elliptic analysis directly on the space $\mathscr{A}$ of all connections (vector potentials), as done by Atiyah and Singer [2]. An important ingredient about G is its integrability criterion, the Wess-Zumino consistency condition. To go from G to the covariant axial anomaly $\tilde{G}$, one can use the explicitly given X operator of Bardeen and Zumino [4].

The present note is motivated by two questions: What is the intrinsic integrability condition for the covariant anomaly $\tilde{G}$? And what is the geometrical interpretation of the aforementioned X operator? Inspired by Atiyah and Singer's success in dealing directly with the geometry of the space $\mathscr{A}$ of all connections, we feel it would be instructive to examine our questions from the viewpoint of presymplectic geometry on $\mathscr{A}$. The abstract summarizes our results.

Our presentation is organized as follows. Section 1 sets up the terminology and notation concerning $\mathscr{A}$ and the group of gauge transformations which acts on it,

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