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On the Stochastic Quantization of Field Theory

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Abstract. We give a rigorous construction of a stochastic continuum $P(\phi)_2$ model in finite Euclidean space-time volume. It is obtained by a weak solution of a non-linear stochastic differential equation in a space of distributions. The resulting Markov process has continuous sample paths, and is ergodic with the finite volume Euclidean $P(\phi)_2$ measure as its unique invariant measure. The procedure may be called stochastic field quantization.

Introduction

Ever since the original work of Glauber [24], there has been much interest in stochastic statistical mechanical models. Such models have been rigorously studied by Holley, Stroock, Faris, Wick, and others [25–30]. The fundamental aim in these works is to obtain (and study properties of) Gibbs states of classical statistical mechanics as limiting distributions of stochastic processes. These processes are sometimes obtained as solutions of non-linear stochastic differential equations of the Langevin type (see later). Let e^{tL} be the associated semi-group, and, starting from an initial state (probability measure) μ_0, let μ_t be the evolved state under the action of the adjoint semigroup acting on the space of measures equipped with the weak * topology. If $\mu_t \to \mu$ in this topology, then μ is the unique equilibrium (invariant) measure. (Sometimes only a subsequence μ_{t_K} converges using weak commpactness criteria.) Let μ be an invariant measure. Then μ is a Gibbs state iff e^{tL} is a selfadjoint contraction on $L^2(d\mu)$. If the invariant measure is unique, the process is ergodic.

In [18], Parisi and Wu proposed such a program for Euclidean quantum field theory. We may call this the method of *stochastic quantization*. This is natural because of the analogy between Euclidean quantum field theory and classical statistical mechanics. Euclidean quantum field theory is described by a probability

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