Commun. Math. Phys. 101, 323–339 (1985)

Communications in Mathematical Physics

Phenomenon of Mobility in Non-Linear Theories*

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Abstract. Homeomorphisms of the unit-sphere of states are studied. Generalizations of the Piron statement and Wigners' theorem are obtained. It is shown that if the semigroup of the unitary operations of quantum theory were extended by introducing any non-linear operation, a *mobility phenomenon* would occur consisting of a possibility of moving any two states to any two surroundings on the unit sphere. For the resulting "non-linear wave packets" the complementarity doctrine would become impossible because of "fluidity" of the space of states under the dynamical transformations.

1. Introduction

In one of its present day views, the theory of dynamical systems deals with the iterations of one single transformation $U: \Phi \to \Phi$ of a certain manifold of states Φ. Within this line of thought some interesting results have been achieved, such as the existence of the "strange attractors," etc. [1–3]. However, the approach based merely on one operation means a simplification of a dynamical theory *ad extremis*. The real physical system is submerged in a variable universe; by modifying it, one can induce many distinct evolution operations. Hence, a complete dynamical theory should deal not with one but with many alternative operations [4–8]. In the "multi-operation dynamics" some new questions arise. One of them is: what kind of transformations of the manifold of states Φ can be basically achieved by manipulating the operations available? [7–9]. In the dynamical theories of quantum mechanical character, the answer is basically known. Here, the evolution operations are unitary transformations of the unit sphere in a Hilbert space. Provided that there are no superselection rules, all the unitary transformations can be principally achieved by employing adequate external fields [7, 10, 11]. A qualitatively new phenomenon arises when the idea of multi-operational dynamics is associated with non-linear evolution equations.

* Supported by COSNET/SEP

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