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Renormalization Theory in Four-Dimensional Scalar Fields (I)

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Abstract. We present a renormalization group approach to the renormalization theory of Φ^4_4, using techniques that have been introduced and used in previous papers and that lead to very simple methods to bound the coefficients of the effective potential and of the Schwinger functions. The main aim of this paper is to show how one can in this way obtain the $n!$-bounds.

1. Introduction

Recently we developed a new technique to construct field theories without cutoffs [1] and our method came to play an important role in several papers, where they have been combined with new brilliant ideas [2, 3].

The possibility of treating situations as complex as that arising in the ultraviolet stability of the Coulomb gas [4] in two dimensions relies on the effectiveness of our method of performing the renormalization which allowed us to avoid completely the consideration of unboundedly large orders of perturbation theory in dealing with the construction of superrenormalizable field theories, in contrast with what was done in the previous breakthrough papers [11].

We shall illustrate in our work how this method can be naturally applied to derive some of the deepest mathematical results of the formal perturbation theory, namely Hepp's theorem [5] and the de Calan, Rivasseau $n!$-bounds [6]. The experts will recognize in the discussion below most of the intriguing difficulties met in [5, 6] and the ideas used to attack them; they will also recognize a somewhat different pattern of solution of the problems and several technical differences which, we hope, make our presentation something more than a rewriting of old results.

We perform our discussion in "coordinate space" rather than in the usual "momentum space" because we think that it is much easier and we study the

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