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# Gauge Choice in Witten's Energy Expression*

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**Abstract.** Witten's equation $\not{D}\psi = 0$ can be interpreted as a gauge fixing condition for classical supergravity. We rigorously prove the existence of asymptotically constant solutions of the more general gauge condition $\not{D}\psi = A\psi$ for almost all endomorphisms $A$ of the spin bundle. Each gives an expression for the gravitational energy similar to Witten's. These include the choice $A = \sqrt{\mathscr{R}}$, which yields the particularly elegant energy expression first noticed by Deser.

## 1. Introduction

Several years ago R. Schoen and S. T. Yau [12] succeeded in proving the Positive Energy Theorem in General Relativity. Shortly thereafter, E. Witten [14] discovered a second proof based on an integration-by-parts formula for the gravitational energy. These breakthroughs stimulated considerable recent work in relativity and have led to a much better understanding of the total mass and energy of isolated gravitational systems in General Relativity.

Witten's proof was to some extent inspired by supergravity, although supergravity plays no direct role in his argument. The exact manner in which Witten's energy expression emerges from supergravity has recently been clarified by Horowitz and Strominger, Deser, Teitelboim, and others (see [13] for references). In this context, the energy formula emerges from the underlying gauge theory of supergravity, and is seen to involve a gauge choice. Specifically, Witten used the Dirac equation $\not{D}\psi = 0$ as a gauge-fixing condition. Many other choices are possible, and each yields an expression for the gravitational energy. These energy expressions display the gauge aspects of supergravity, yet do not involve the anticommuting fields nor supersymmetry which characterize the full supergravity theory.

This paper establishes several specific theorems about partial differential equations. These are intended to emphasize and clarify the gauge aspects of Witten's energy formula. Theorem 1 is a precise statement showing that an appropriate form of Witten's energy expression is valid even when Einstein's equations are not satisfied and when the spinor field does not satisfy the Dirac equation. Our main

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