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Long Range Order in the Anisotropic Quantum Ferromagnetic Heisenberg Model

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Abstract. We study the anisotropic quantum mechanical ferromagnetic Heisenberg model. By anisotropic we mean that the x and y exchange constants are equal but smaller than the z exchange constant. We show that for any amount of anisotropy there is long range order in two or more dimensions at low enough temperature. We also develop a convergent low temperature expansion and use it to prove exponential decay of the truncated correlation functions.

1. Introduction

The Hamiltonian of the quantum mechanical ferromagnetic Heisenberg model is

$$-\sum_{\langle rs\rangle}(\alpha_x\sigma_r^x\sigma_s^x+\alpha_y\sigma_r^y\sigma_s^y+\alpha_z\sigma_r^z\sigma_s^z).$$

The isotropic Heisenberg model is obtained by taking $\alpha_x=\alpha_y=\alpha_z$. We will study the anisotropic case $\alpha_x=\alpha_y=\alpha$, $\alpha_z=1$ with $\alpha<1$. Our main result, Theorem 2.1, is that for two or more dimensions and any $\alpha<1$, there is long range order (LRO) at sufficiently low temperature. By LRO we mean that $\langle\sigma_i^z\sigma_j^z\rangle$ is bounded away from zero uniformly in i and j. We use free boundary conditions to define the Gibbs state $\langle\ \rangle$. We will also show that a polymer expansion for the model converges for sufficiently low temperature. This yields more detailed information than LRO, e.g., the truncated correlation functions decay exponentially (see Theorem 2.2).

Ginibre [8] and Robinson [12] proved the existence of LRO at low temperature for sufficiently small α. Kirkwood [9] and Thomas and Yin [14, 15] proved LRO at low temperature in similar models. Fröhlich and Lieb [6] showed that if reflection positivity is true for the model then there is LRO for any $\alpha<1$. However, Speer [18] has shown that reflection positivity is not always true for the model. For the classical Heisenberg model Malyshev [10] proved the occurrence of LRO for any $\alpha<1$. This result was improved by Bricmont and Fontaine [17].

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