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Oscillator Representations of the 2D-Conformal Algebra

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Abstract. We display irreducible representations of the Virasoro algebra (group of diffeomorphisms of the circle) for any value of the central charge c (central extension defined by a cocycle) and of the highest weight ε, where the Kač determinants do not vanish. The construction is done in terms of a simple bosonic free field. The unitarity of the representation is discussed, and it is realized with non-trivial hermiticity properties of the free field if $\varepsilon<(c-1)/24$. In the particular case of the central charge ($c=\frac{1}{2}$) corresponding to the Ising model, the three unitary irreducible representations ($\varepsilon=0, \frac{1}{16}, \frac{1}{2}$) are realized in terms of the anticommuting oscillators of the free fields of the Neveu-Schwarz-Ramond model.

A decade after they were introduced in string theories [1], the representations of the conformal group in two dimensions are the subject of a growing interest in physics as well as in mathematics. In this communication, we discuss a general form for the irreducible representations of the associated Lie algebra with central charge, the so-called Virasoro algebra:

$$[L_n, L_m]=(n-m)L_{n+m}+\frac{c}{12}(n^3-n)\delta_{n,-m}, \tag{1}$$

with the hermiticity condition

$$L_n=L^+_{-n}, \tag{2}$$

where n, m are integers, c is a real number. For a given c, the irreducible representations are characterized by the ground state (highest weight vector) $|\varepsilon\rangle$ such that

$$L_n|\varepsilon\rangle=0, \quad n>0, \qquad L_0|\varepsilon\rangle=\varepsilon|\varepsilon\rangle. \tag{3}$$

All states can, in principle, be obtained by repeated applications of $L_{-n}(n>0)$ to $|\varepsilon\rangle$. Past experience has shown that this way of building representations is often