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Some Comments on the ADHM Construction in $4k$ Dimensions

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Abstract. A class of completely solvable gauge field equations is investigated. These equations are shown to be closely analogous to the four-dimensional self-duality equations. A similar geometrical interpretation is exhibited, and a generalisation of the ADHM construction is shown to provide solutions.

1. Introduction

Recently there has been considerable interest shown in field theories defined on space times of dimension greater than four. Much of the activity has been in the area of Kaluza–Klein theories elaborating the ideas of Cremmer and Scherk, Witten and many others [1]. However, there has also been some investigation of the possibility of extending the idea of "self-duality" and applying it to pure Yang–Mills theories in higher dimensions. In other words, equations linear in the gauge field strengths $F_{\mu\nu}, \mu, \nu = 1 \ldots D$, are sought which will, as a consequence of the Bianchi identities imply the full second order gauge field equations,

$$D^\mu F_{\mu\nu} = 0. \tag{1}$$

Linear relations of this type were studied in Ref. [2].

Ward [3] has pointed out that amongst these linear relationships implying the full field equations there are some, but by no means all, which arise as integrability conditions for certain sets of first order differential equations. Ward's first order equations generalise the pair introduced by Belavin and Zahkarov [4], whose integrability condition is the usual four dimensional self-duality equation,

$$F_{\mu\nu} = \pm {}^*F_{\mu\nu}, \quad {}^*F_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}. \tag{2}$$

More explicitly, their first order equations may be written

$$e_\mu^\dagger \pi D^\mu \psi = 0, \tag{3}$$

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