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Averaging Operations for Lattice Gauge Theories*

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Abstract. Usually renormalization group transformations are defined by some averaging operations. In this paper we study such operations for lattice gauge fields and for gauge transformations. We are interested especially in characterizing some classes of field configurations on which the averaging operations are regular (e.g., analytic). These results will be used in subsequent papers on the renormalization group method in lattice gauge theories.

Introduction

In Wilson's approach to renormalization group transformations [9, 10] for lattice gauge systems, it is necessary to define an operation of taking an average of field configurations over subdomains of a lattice. These subdomains are usually some simple subsets, for example cubes of a fixed size, or sums of several such cubes. In this paper we will study one such definition of an averaging operation. This operation will be used in other papers on gauge field theories.

Let us introduce some definitions and notations. We will be very sketchy because these definitions have already appeared several times in the earlier papers [1, 2] of the author and we refer the reader to these papers, especially to [2], for more detailed explanations. We consider a subdomain Ω of the lattice ηZ^d with a lattice spacing η. A sequence of sets $\Omega^{(j)}$ is defined as the intersections

$$\Omega^{(j)} = \Omega \cap L^j \eta Z^d , \tag{1}$$

where L is a fixed integer, $L > 1$. For a point $y \in L^n \eta Z^d$ (or any lattice δZ^d), we define a block of an order j as the cube

$$B^j(y) = \{x \in L^{-j} L^n \eta Z^d : y_\mu \leqq x_\mu < y_\mu + L^n \eta,\ \mu = 1, \dots, d\} \tag{2}$$

(or the corresponding cube with $L^n\eta$ replaced by δ). We will omit the subscript j if $j = 1$. For a subset $\Lambda \subset L^n \eta Z^d$ (or $\subset \delta Z^d$), we define

$$B^j(\Lambda) = \bigcup_{y \in \Lambda} B^j(y) \subset L^{-j} L^n \eta Z^d \quad (\text{or } \subset L^{-j} \delta Z^d) . \tag{3}$$

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