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The Asymptotic Higgs Field of a Monopole

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Abstract. A simple formula is computed for the asymptotic Higgs field of an SU(2) monopole. This formula is derived from the twistor description of monopoles, and is applied to the study of boundary behaviour. It is found to be harmonic, and to have as its natural domain of definition a branched covering of $\mathbb{R}^3$. Explicit formulae are given in simple cases.

1. Introduction

The purpose of this paper is to compute a simple asymptotic formula for the Higgs field of an SU(2)-monopole, valid up to exponentially decreasing terms: the "algebraic" part of the Higgs field [11]. This formula is derived from the twistor description of monopoles, and is intimately linked with the geometry of the monopole's spectral curve [3], as follows: a point in $\mathbb{R}^3$ corresponds to a section of $T(\mathbb{P}_1(\mathbb{C}))$ over $\mathbb{P}_1(\mathbb{C})$; this section intersects the spectral curve in $2k$ points; choosing k out of the $2k$ (asymptotically this choice is canonical), one evaluates the slope of the curve at these points, and sums, adding 1, to obtain the asymptotic norm of the Higgs field. This expression is harmonic, and is seen, in particular cases, to arise from a "charge distribution" on a union of compact disk-like surfaces in $\mathbb{R}^3$. We also apply the formula to give a direct proof that the boundary conditions of a monopole are satisfied when the spectral curve satisfies the conditions given in [3].

Let P be a principal SU(2)-bundle over $\mathbb{R}^3$, p its associated su(2) bundle, Φ a section of p, ∇ a connection on P, with F its associated curvature. The couple (∇, Φ) is an SU(2) monopole if the following conditions are satisfied:

1) $*F = \nabla\Phi$, where $*$ is the Hodge star operator on two-forms over $\mathbb{R}^3$. (Bogomolny equations [1])

2) The boundary conditions, as $r \to \infty$:

a) $|\Phi| = 1 - k/2r + O(r^{-2})$,
b) $\partial|\Phi|/\partial\Omega = O(r^{-2})$,
c) $|\nabla\Phi| = O(r^{-2})$.

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