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On the Modular Structure of Local Algebras of Observables

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Abstract. An asymptotic expression for the modular operators of local algebras of observables in quantum field theory is given. In an asymptotically scale invariant theory this leads to an identification of the spectra of all modular operators with $\mathbb{R}_+$. So in this case the local algebras are algebras of type III, and only factors of type III_1 can occur in a central decomposition.

1. Introduction

A characteristic feature of relativistic quantum field theory is the existence of vacuum fluctuations for all local observables. Actually, the vacuum state, restricted to a bounded region $\mathcal{O}$, has many features of an equilibrium state at nonzero temperature. Mathematically, this analogy has been made precise by the Tomita-Takesaki theory [1]. The vacuum vector Ω is cyclic and separating for each local algebra $\mathfrak{A}(\mathcal{O})$ according to the Reeh-Schlieder theorem [2]. Therefore, the theory of Tomita and Takesaki implies the existence of a positive invertible operator $\Delta_{\mathcal{O}}$ such that $\Delta_{\mathcal{O}}^{it}\mathfrak{A}(\mathcal{O})\Delta_{\mathcal{O}}^{-it}\subset\mathfrak{A}(\mathcal{O})$ for all $t\in\mathbb{R}$, and the vacuum is an equilibrium state with temperature 1 if $\Delta_{\mathcal{O}}^{-it}$ is considered as the time translation operator.

It is not clear from the beginning whether the "dynamics" which generates these time translations has a physical interpretation. There is an important special case, where such an interpretation is possible. Bisognano and Wichmann [3] have shown that for the wedge $W=\{x\in\mathbb{R}, |x^0|<x^1\}$ the operators Δ_w^{-it} coincide with the unitary operators $U(\Lambda_{2\pi t})$ which implement the Lorentz transformation

$$\Lambda_{2\pi t}=\begin{pmatrix}\cosh 2\pi t & \sinh 2\pi t & & \\ \sinh 2\pi t & \cosh 2\pi t & & 0 \\ & & 1 & \\ & 0 & & 1\end{pmatrix}. \tag{1.1}$$

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