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Infinitely Divisible Distribution Functions of Class L and the Lee–Yang Theorem

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Abstract. It is shown that the free-energy density of a large class of ferromagnets satisfying the Lee–Yang property is to be connected with the limit characteristic function of a suitably renormalized sum of independent and non-identically distributed random variables. Using the canonical representation formulae of such characteristic functions, various chains of inequalities are derived for the Ursell functions.

1. Introduction

In spite of considerable efforts, only a few models have been exactly solved in statistical mechanics. It is therefore of great interest to find general properties of the free-energy density or other related variables at the thermodynamic limit. An important contribution along this direction was achieved many years ago by Lee and Yang [1, 2]. If $Z_N(\beta, \beta h)$ denotes the partition function of a d-dimensional Ising ferromagnet with pairwise interactions (as usual, β is the reciprocal temperature, h is the external field and N is the number of spins), they proved that the zeros of $Z_N(\beta, \beta h)$ as a function of $z = \exp(2\beta h)$ all lie on the unit circle in the complex z plane.

A number of interesting results have been derived since using various forms of the Lee–Yang theorem (e.g. [3–5]). Recently, J. De Coninck and Ph. de Gottal [6] established a connection between the Lee–Yang theorem and the set of infinitely divisible distribution functions (see below). This leads to various inequalities for the Ursell functions of finite systems.

In this paper, we extend this analysis by showing that such a connection also holds when the thermodynamic limit is considered. Before presenting our results, let us recall some terminology ([8, 9]).

Let $(X_{ij})_{i\geqq 1, 1\leqq j\leqq k_i}$ be a double sequence of random variables that are mutually independent for a fixed i and subject to the condition of infinite smallness, i.e. for every $\varepsilon > 0$.

$$\max_{1\leqq k\leqq k_n} P\{|X_{nk}| \geqq \varepsilon\} \to 0 \quad \text{as } n \to \infty. \tag{1}$$