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The KAM Theory of Systems with Short Range Interactions, I*

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Abstract. The existence of quasiperiodic trajectories for Hamiltonian systems consisting of long chains of nearly identical subsystems, with interactions which decay rapidly with increasing distance between the interacting components, is studied. Such models are of interest in statistical mechanics. It is shown that nonergodic motions persist for much larger perturbations than prior work indicated. If the number of degrees of freedom of the system is N, the allowed perturbation decreases only as an inverse power of N, as the number of degrees of freedom increases, rather than the inverse power of $N!$ which previous estimates yielded.

1. Introduction

The Kolmogorov, Arnol'd, Moser (KAM) theory [15, 1, 16] proves that "small" perturbations of integrable Hamiltonian systems possess "large" sets of initial conditions for which the trajectories remain quasiperiodic. In this paper we discuss how the "strength" of the allowed perturbation varies with the number of degrees of freedom, N, in the system. (We give precise meanings to the words in quotation marks below.) Classical estimates for a general analytic perturbation of strength ε_0 require

$$\varepsilon_0 < C(N!)^{-\alpha} \tag{1.1}$$

to ensure that the theory applies. Here C is a constant depending on all the parameters of the system except N, and [11] gives a value of $\alpha = 31$.

Recent numerical experiments [4, 6, 3, 10] indicate that at least in systems with short range interactions, perturbations much larger than those permitted by (1.1) still give rise to quasiperiodic motion. In the present paper we initiate a study of such

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