Commun. Math. Phys. 96, 285—309 (1984)

Communications in
Mathematical
Physics

Geometry of $N = 1$ Supergravity (II)

A. A. Rosly and A. S. Schwarz
Moscow Physical Engineering Institute, Kashirskoe Shosse 1, Moscow M-409, USSR

Abstract. The supergravity torsion and curvature constraints are shown to be a particular case of constraints arising in a general geometrical situation. For this purpose, a theorem is proved which describes the necessary and sufficient conditions that the given geometry can be realized on a surface as one induced by the geometry of the ambient space. The proof uses the theory of nonlinear partial differential equations in superspace, Spencer cohomologies, etc. This theorem generalizes various theorems, well known in mathematics (e.g., the Gauss–Codazzi theorem), and may be of its own interest.

1. Introduction

In a previous paper [1] we studied the geometry of various superspace formulations of $N = 1$ supergravity. We considered a well-known family of supergravity models labelled by a parameter ζ, and found that different approaches to supergravity are connected with a general geometrical problem. Suppose one has a space endowed with a fixed geometry of some type. Then, given some surface in this space, one can define the internal geometry of the surface, induced on it by the geometry of the ambient space. The relevant general definition of induced geometry uses the language of G-structures (see refs. [1, 2]; the present paper is not completely self-contained, but uses the notations and conventions of ref. [1]).

In this paper we prove a theorem (Sect. 2) about the necessary and sufficient conditions that the given G'-structure on a manifold can be realized on some surface as one induced by the trivial G-structure in $\mathbb{R}^N$. In general, this problem amounts to the question whether a certain system of nonlinear partial differential equations has a solution. The theorem describes the conditions of the formal integrability (Appendix B) for that system in a convenient form of constraints on the internal geometry. There is, in general, a chain of integrability conditions of increasing orders. The number of non-trivial ones, which is always finite, is controlled by certain Spencer cohomologies (Appendix A) related to the problem.

Thus, we shall see that a G'-structure corresponding to induced geometry is not arbitrary, but satisfies certain constraints. It turns out that the supergravity torsion and curvature constraints are just of that nature. In refs. [1, 2] it was shown that in