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On the Existence of Thermodynamics for the Random Energy Model

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Abstract. Derrida's random energy model is considered. Almost sure and L^p convergence of the free energy at any inverse temperature β are proven. Rigorous upper and lower bounds to the finite size corrections to the free energy are given.

Introduction

The Random Energy (R.E.) Model has been introduced by Derrida [I, II] as a simplified version of the mean field Sherrington-Kirkpatrick (S.K.) model [III] of a spin glass.

Both in the S.K. and in the R.E. models, the energies associated to each spin configuration in the volume N, are gaussian random variables with mean zero and covariance N.

In the S.K. model, we have an explicit microscopic hamiltonian, where the couplings are assumed to be independent gaussian normalized random variables so that the energies turn out to be dependent, whereas in the R.E. model, the microscopic hamiltonian is not specified and the energies are supposed to be independent random variables with the proper normalization. Thus the R.E. partition function has the following expression:

$$Z_N = \sum_{i=1}^{2^N} \exp \beta \sqrt{N} X_i ; \qquad X_i \in \mathcal{N}(0,1) . \tag{I.1}$$

In a recent paper, Eisele [IV] studied the R.E. model in a slightly more general situation by means of the theory of large deviations. He rigorously proved that the quenched free energy converges as $N \to \infty$, to a function $F(\beta)$ whose second derivative is discontinuous at $\beta = \beta_c \equiv \sqrt{2 \log 2}$ (third order phase transition). He was able to prove the almost sure convergence of the free energy only for $\beta \leqq \beta_c$, whereas he showed the stochastic convergence for any β. In the present paper we study the R.E. model by means of quite elementary techniques and establish the

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