Commun. Math. Phys. 96, 59–95 (1984)

Communications in Mathematical Physics
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Generation of Vorticity Near the Boundary in Planar Navier-Stokes Flows*

G. Benfatto and M. Pulvirenti

Dipartimento di Matematica dell'Università di Roma La Sapienza, Piazza A. Moro, I-00100 Roma, Italy

Abstract. We construct the solutions of the planar Navier-Stokes flow for a viscous incompressible fluid in the half-plane, by means of a boundary layer equation describing the production of vorticity on the boundary. Regularity properties are also discussed.

0. Introduction

The time evolution of a viscous incompressible fluid is usually described by the Navier-Stokes equations

$$\begin{cases} \dfrac{\partial u}{\partial t} + (u \cdot \nabla) u = \nu \Delta u - \nabla p + g, & (1) \\ \nabla \cdot u = 0, & (2) \end{cases}$$

where

$$u : D \times [0, T] \to \mathbb{R}^d, \qquad d = 2, 3, \qquad T > 0, \tag{3}$$

denotes the velocity field, $D \subset \mathbb{R}^d$ (d the dimension of the physical space) is an open region with smooth boundary in which the fluid is confined, $\nu > 0$ is the viscosity coefficient, $p : D \times [0, T] \to \mathbb{R}^1$ is the pressure and, finally, $g : D \times [0, T] \times \mathbb{R}^d$ is an external force acting on the system.

Equations (1) and (2) describe the balance of momentum and conservation of mass, respectively. For simplicity the density is assumed to be one.

We are interested in the initial boundary value problem (ibvp) associated with (1) and (2). This means that we have to specify the initial value of u, i.e.

$$u(x, 0^+) = u_0(x), \qquad x \in D, \tag{4}$$

* Research partially supported by Ministero della Pubblica Istruzione and CNR-GNFM