

The $1/N_F$ Expansion of the γ and β Functions in Q.E.D.

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Abstract. The Callan-Symanzik γ - and β -functions are calculated analytically for Q.E.D. in the limit of a large number of leptons ($N_F \rightarrow \infty$) up to terms of order $1/N_F$ inclusive. We give closed analytic expressions for the coefficients of these terms in their series expansion in powers of $K \equiv \alpha N_F/\pi$. We have been able to sum these series and to obtain some striking results.

1. Introduction

Apart from the first few perturbative terms of the renormalization group functions, not much more is known about them. To our knowledge in four-dimensional field theories one has gone beyond that only in $\lambda\Phi^4$ and in Q.E.D. in the limit of a large number of fermions, $N_F \rightarrow \infty$. In $\lambda\Phi^4$ Khuri [1] has been able to find a zero of the Callan-Symanzik β -function [2] by using a Borel resummation technique, and in Q.E.D. ($N_F \rightarrow \infty$) Coquereaux [3] has computed the first nontrivial coefficient in the $1/N_F$ expansion of the same function. In a recent paper [4] we have computed the Callan-Symanzik γ -function [2], which governs the dependence on the renormalization point of the renormalized fermion mass, in Q.E.D. ($N_F \rightarrow \infty$). The main results obtained there were:

i) The numerical computation of the first 19 terms strongly suggests a series with a finite radius of convergence. This is not what one expects in four-dimensional field theories.

ii) There are at least two zeros within the region of convergence.

iii) The analytical computation of the first 7 terms clearly hints on a factorization of the series into two factors. These are two series, one of them of only rational coefficients, and the other of coefficients which are sums of products of Riemann ζ -functions. To our knowledge this is the first time such a factorization has been found in field theory.

While trying to extend these results to the Callan-Symanzik β -function in Q.E.D. ($N_F \rightarrow \infty$) we have found that the results of our previous work can be considerably improved and extended to the calculation of the β -function. The main results of the present paper are: