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Exact Theory of the (Einstein) Gravitational Field in an Arbitrary Background Space-Time

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Abstract. The Lagrangian based theory of the gravitational field and its sources at the arbitrary background space-time is developed. The equations of motion and the energy-momentum tensor of the gravitational field are derived by applying the variational principle. The gauge symmetries of the theory and the associated conservation laws are investigated. Some properties of the energy-momentum tensor of the gravitational field are described in detail and the examples of its application are given. The desire to have the total energy-momentum tensor as a source for the linear part of the gravitational field leads to the universal coupling of gravity with other fields (as well as to the self-interaction) and finally to the Einstein theory.

1. Introduction

Investigations on general relativity (GR) are frequently being carried out under the assumption that there exists some background space-time. Problems of post-Newtonian equations of motion, generation and propagation of weak gravitational waves, quantization of weak gravitational field apply the notion of flat background space-time [1–3]. A curved background space-time (especially, cosmological and black hole geometries) can be used when the propagation and amplification of perturbations or vacuum polarization and particle creation effects are considered. The "background field method" is developed for treating the various quantum fields at the classical background [4]. The notion of background space-time, primarily flat, is also invoked when interpreting the solutions of GR equations.

Investigations of this type are being carried out, as a rule, in the linear approximation, without taking into account the "back reaction" of perturbations, or, in a better case, by successive approximations. Many additional constructions are used, like averaging over space-time volumes, specific choice of coordinate conditions, asymptotically Cartesian coordinates, etc. Unfortunately, it is not