Commun. Math. Phys. 94, 317–329 (1984)

Communications in Mathematical Physics

Breaking and Disappearance of Tori

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Abstract. A mechanism is illustrated which can cause a torus to disappear in dissipative differential equations. Three different examples give evidence that a collision with a neighbouring unstable periodic orbit, possibly preceded by a transition into a weakly chaotic attractor, causes the sudden destruction of a torus.

1. Introduction

An open problem in dissipative dynamical systems is a satisfactory understanding of the different possible transitions away from quasiperiodic flow on a two-torus. Those transitions which lead to a turbulent flow on a strange attractor are of particular interest.

In a fundamental paper by Ruelle and Takens [1], a theoretical study of differential equations shows that a quasiperiodic motion with two independent frequencies, when followed by one with three independent frequencies, leads under generic conditions to the appearance of a strange attractor. Support of this mechanism for turbulence has been found experimentally by Gollub and Benson [2] and numerically by Yahata [3]. There are also, however, examples of transitions from a two-torus directly to a strange attractor. This has been observed in experiments (see, for example, again [2]) and, with different features, in a number of numerical studies of nonlinear differential equations. Curry [4] has found two symmetrically located tori which become unstable and give rise to a wide chaotic attractor surrounding them both. Arneodo et al. [5] and Franceschini [6] have shown that chaos can be preceded by a cascade, possibly finite, of period-doublings of a torus. Schreiber and Marek [7] have given an example in which a torus becomes a strange attractor producing some kind of foldings or wrinkles. Riela [8] has exhibited a picture showing a torus which, quite evidently on a section, disappears after development of corners.

Other approaches to the problem are possible. Shenker [9], Feigenbaum et al. [10], and Rand et al. [11, 12] have considered mappings of the circle into itself.